Placement Test Review Materials for Elementary Algebra Review
To The Student

This workbook will provide a review of some of the skills tested on the COMPASS placement test. Skills covered in this workbook will be used on the Algebra section of the placement test.

This workbook is a review and is not intended to provide detailed instruction. There may be skills reviewed that you will not completely understand. Just like there will be math problems on the placement tests you may not know how to do. Remember, the purpose of this review is to allow you to “brush-up” on some of the math skills you have learned previously. The purpose is not to teach new skills, to teach the COMPASS placement test or to allow you to skip basic classes that you really need.

The Algebra test will assess your mastery of operations using signed numbers and evaluating numerical and algebraic expressions. This test will also assess your knowledge of equations solving skills used to solve 1-variable equations and inequalities, solve verbal problems, apply formulas, solve a formula for a particular variable, and find “k” before you find the answer. You will also be tested on your knowledge of exponents, scientific notation, and radical expressions, as well as operations on polynomials. Furthermore, other elementary algebra skills that will be on the test include factoring polynomials, and using factoring to solve quadratic equations and simplify, multiply, and divide rational expressions.
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Answer Key 34
Using Order of Operations

Rules: 1. Do the operation inside the parentheses, braces, brackets, or absolute value first. If more than one, do the innermost parentheses first.
2. Do the exponents next.
3. Multiply and Divide from left to right.
4. Add and Subtract from left to right.

Examples: 

\[ 5 + 3(8 - 9) - 7 = \]
\[ 5^2 - 3(7 + 2) \div 27 = \]
\[ 5 + 3(8 + -9) + -7 = \]
\[ 5^2 + -3 (9) \div 27 = \]
\[ 5 + 3(-1) + -7 = \]
\[ 25 + -3(9) \div 27 = \]
\[ 2 + -7 = -5 \]
\[ 25 + -27 \div 27 = \]
\[ 25 + -1 = 24 \]

WATCH OUT!
In an expression like \( \frac{3(-2) - 5}{-7 + 2(-2)} \), the fraction bar acts like a grouping symbol. Simplify the numerator and denominator separately, and then divide.

Practice:

1. Simplify: \( 8 \div 2^3 - 12 \div 2^2 \)
2. Simplify: \( 2 \cdot 4^2 - 6 \div 2 - 9 \cdot 4 \)
3. Evaluate: \( 14 - 27 \div (3 - 6)^2 - 3 \)
4. Evaluate: \( \frac{36(-6)}{4(2-5)^2} \)
Simplifying Complex Fractions

Simplify the numerator and the denominator first, and then divide the numerator by the denominator.

\[
\frac{3 + \frac{9}{20}}{10 + \frac{20}{20}} = \frac{15 + \frac{18}{20}}{20} = \frac{33}{20} \cdot \frac{2}{5} = \frac{33}{50}
\]

Practice:

1. Simplify: \(\frac{-4}{1 + \frac{3}{3} + \frac{8}{3}}\)

2. Simplify: \(\frac{-1 - \frac{3}{5}}{-1 + \frac{1}{10}}\)

3. Evaluate: \(\frac{2 - \frac{1}{3}}{\frac{4}{5} - \frac{1}{7}}\)

4. Evaluate: \(\frac{2 - \frac{3}{\left(\frac{1}{2}\right)^2}}{}\)
Evaluating Algebraic Expressions

Substitute the value(s) for the variable(s). If the value is a negative number, use a parentheses around the number. This will help prevent mistakes with negative signs. Use order of operations to simplify.

Examples:

Evaluate: $-4(a - 2b) - bc$ for $a = 3$, $b = -2$ and $c = 4$.

$-4(a - 2b) - bc = -4(3 - 2(-2)) - (-2)4$

$= -4(3 - (-4)) - (-8)$

$= -4(3 + 4) + 8$

$= -4(7) + 8$

$= -28 + 8$

$= -20$

Find the value of $3x^2 - x + 2$ when $x = -2$.

$3x^2 - x + 2 = 3(-2)^2 - (-2) + 2$

$= 3(4) + 2 + 2$

$= 12 + 2 + 2$

$= 16$

Practice:

1. Evaluate $a^2 - b^2$ when $a = 4$ and $b = -3$.  

2. Evaluate: $\frac{-2}{3}x + \frac{1}{2}(xy - 4x)$ when $x = -6$ and $y = 3$.

3. Evaluate $-2a^2 + 3b - c$ when $a = -3$, $b = 2$, and $c = 4$.

4. Evaluate: $\frac{1}{3}m^2 - \frac{1}{4}n^2$ when $m = 3$ and $n = 8$. 

5. Find the value of $-x^5 + 4x^2 - 9x - 7$ for $x = -1$.  

6. Evaluate the following polynomial for $y = -1$.  
   
   $3y^2 - y + 5$
Using Formulas

Read the problem carefully.
Identify the known and unknown variables.
Write a formula that relates the variable in the problem
Substitute the known values for the variables.
Solve the equation and answer the question.

Example:
Find the length of weather stripping needed to put around a rectangular window that is 9 ft. wide and 22 ft. long. The perimeter of a rectangle is found by using the formula: \( P = 2L + 2W \), \( L = \) length and \( W = \) width.

Known variables: \( W = 9, L = 22 \)  
Unknown variable: \( P \)

\[
P = 2(22) + 2(9) \]
\[
P = 44 + 18 \]
\[
P = 62 \text{ ft.} \]

Practice:
1. Find the length of aluminum framing needed to frame a picture that is 6 ft. by 4 ft. The formula for the perimeter of a rectangle is \( P = 2L + 2W \), where \( L = \) length and \( W = \) width.

2. Find the approximate length of a rubber gasket needed to fit around a circular porthole that has a 20-inch diameter. The formula for the circumference of a circle is \( C = \pi \cdot d \), where \( d = \) diameter. Use \( \pi = 3.14 \).
3. A room 11 ft. by 15 ft. is to be carpeted. Find the number of square yards of carpet needed \((9\text{ft}^2 = 1\text{yd}^2)\). Round to the nearest tenth.

4. The surface area of a right circular cylinder can be found by using the formula: \(SA = 2\pi rh + 2\pi r^2\). Use the formula to find the surface area of an oil drum whose radius is 4 ft. and whose height is 5 ft. Use \(\pi = 3.14\).
Solving 1-Variable Equations

An equation is a mathematical sentence that involves an equal sign.
Examples of equations in one variable are:

\[ 3x - 6 = 9 \]
\[ 2x + 5x - 8 = 5x - 3 \]
\[ 3(x - 2) = 5x + 6 \]

The following steps are used to solve an equation:
1. Remove parentheses and combine like terms on each side of the “=”.
2. Use the addition property to get all terms with variables on one side of the equation and all other terms on the other side. (Remember, if you add a term to one side of the equation, then you must add the same term to the other side to keep the equation balanced or true.
3. Combine like terms on each side of the equation.
4. Use the multiplication or division property to solve for the variable. Remember to multiply or divide both sides in order to keep the equation balanced or true.
5. Check your solution for the variable by substituting into the original equation and simplifying. The result should be a true statement.

Examples:

Solve: \[ 3x - 8 = 4 \]
\[ 3x + -8 = 4 \]
\[ 3x + -8 + 8 = 4 + 8 \]
\[ 3x = 12 \]
\[ \frac{3x}{3} = \frac{12}{3} \]
\[ x = 4 \]

Solve: \[ 3(x + 5) + 2x = 6x + 10 \]
\[ 3x +15 +2x = 6x + 10 \]
\[ 5x + 15 = 6x + 10 \]
\[ 5x + -6x + 15 = 6x + -6x + 10 \]
\[ -1x + 15 = 10 \]
\[ -1x + 15 + -15 = 10 + -15 \]
\[ -1x = -5 \]
\[ -1 \cdot -1 \]
\[ x = 5 \]

Practice:
1. Solve: \[ 5x - 5 = 3x + 7 \]
2. Solve: \[ 2x - 11 = -3x + 9 \]
3. Solve: $5x + 7 + 3x = 47$

4. Solve: $4y + 6 + y = 11$

5. Solve: $\frac{5}{8}x - 3 = -5$

6. Solve: $\frac{1}{3}x + 5 = 2$

7. Solve: $2x - 3(2x - 5) = 7 - 2x$

8. Solve: $3x + 4(2x - 7) = 9x - 22$
Solving 1-Variable Inequalities

To solve an inequality, you follow the same rules that you use for solving an equation **except**: When you multiply or divide both sides by a negative number, you must change the inequality sign.

Examples:

Solve: 2x + 6 > 4

\[
egin{align*}
2x + 6 + (-6) &> 4 + (-6) \\
2x &> -2 \\
2x > -2 \\
x &> -1
\end{align*}
\]

(did not change > sign)

Solve: -3x + 8 > -4

\[
egin{align*}
-3x + 8 + (-8) &> -4 + (-8) \\
-3x &> -12 \\
\frac{-3x}{-3} &< \frac{-12}{-3} \\
x &< 4
\end{align*}
\]

(did change > to < because I divided by a negative number)

Practice:

1. -5x + 2 < -18

2. 3x - 2 > 13

3. Solve: 5x - 1 < -9

4. Solve: 2 - 7x < 16
Writing Verbal Expressions as Equations and Algebraic Expressions

Words that mean add:
- plus
- total
- sum
- increased by
- greater than
- more than

Words that mean subtract:
- minus
- reduced by
- fewer
- difference between
- decreased by
- less than

Words that mean multiply:
- times
- product of
- of
- total (sometimes)

Words that mean divide:
- quotient of
- divisor
- dividend
- ratio
- parts

Examples:

Translate into an algebraic expression or equation:
- three more than twice a number: \(2n + 3\)
- eight less than 1/3 of a number: \(\frac{1}{3}n - 8\)
- four times a number increased by 6: \(4n + 6\)
- the product of a number and 5: \(5n\)

Translate into an equation or inequality. Then solve:

Four more than three times a number equals 8.
\[
3n + 4 = 8
\]
\[
3n + 4 + (-4) = 8 + (-4)
\]
\[
3n = 4
\]
\[
\frac{3n}{3} = \frac{4}{3}
\]
\[
x = \frac{4}{3}
\]

The difference between twice a number and 3 is greater than 15.
\[
2x - 3 > 15
\]
\[
2x + (-3) + 3 > 15 + 3
\]
\[
2x > 18
\]
\[
\frac{2x}{2} > \frac{18}{2}
\]
\[
x > 9
\]
Practice:
1. Translate “six less than the product of n and negative nine” into an algebraic expression:

2. Write “the quotient of ten and the sum of a number and 7” as an algebraic expression.

3. Thirty gallons of sugar were poured into two different sized containers. If g gallons were poured into the larger container, write an expression for the number of gallons poured into the smaller container in terms of the number of gallons poured into the larger container.

4. John bought a rectangular lot. The length of the lot is 25 ft. more than four times the width of the lot. If w is the width of the lot, write an expression that gives the length of the lot?

5. The equation for “five less than four times a number is equal to three” is:

6. Translate “the difference between ten times a number and three times the number is twenty-eight” into an equation and solve.

7. Find four consecutive odd integers whose sum is between 30 and 50.

8. Company A rents a car for $12 a day and 10 cents for each mile driven. Company B rents cars for $23.50 per day with unlimited mileage. Write the inequality representing Company A’s car costs less than Company B’s car.
## Solving an Equation for a Particular Variable

Solving for a particular variable means; getting the particular variable on one side of the equal sign by itself. To do this you should:

1. Remove all grouping symbols.
2. Use equation solving techniques to get terms with the particular variable on one side and all other terms on the other side of the equal sign.
3. Factor out the variable if you have more than one term with the particular variable in it.
4. Divide both sides by the coefficient of the particular variable.

### Examples:

**Solve:** \( P = 2(L + W) \) for \( W \)

\[
P = 2L + 2W \\
\frac{P - 2L}{2} = \frac{2W}{2} \\
W = \frac{P - 2L}{2}
\]

**Solve:** \( a + 3x = c(x + 2) + x \) for \( x \)

\[
a + 3x = cx + 2c + x \\
3x - x - cx = 2c - a \\
2x - cx = 2c - a \\
x(2 - c) = 2c - a \\
x = \frac{2c - a}{2 - c}
\]

### Practice:

1. **Solve:** \( P = \frac{R - C}{a} \) for \( a \)

2. **Solve:** \( k = tr - tv \) for \( t \)

3. **Solve** \( R = \frac{aB - T}{c} \) for \( a \)

4. **Solve** \( S = \frac{R + T}{2} \) for \( R \)

5. If \( as + b - c = 0 \), then \( b = ? \)
Solving Problems that Involve a Second Variable “k”

Find the value of “k” first using the values given for the other variables, then find the value of the variable in the question using the value of “k” that you found.

Example: If \( x = 3 \) and \( y = kx + 7 \), then \( y = 13 \). Find \( y \) when \( x = 6 \).
First, find the value of “k”, using \( y = 13 \) when \( x = 3 \).

\[
13 = k(3) + 7
\]
\[
13 + (-7) = 3k
\]
\[
6 = 3k
\]
\[
\frac{6}{3} = \frac{3k}{3}
\]
\[
k = 2
\]
Then find \( y \) when \( x = 6 \).

\[
y = 2(6) + 7
\]
\[
y = 19
\]

Practice:
1. If \( 3x = k \) and \( k = z + 1 \), what is the value of \( x \) when \( z = 29 \)?

2. If \( y = x^2 + kx + 8 \) and \( y = 0 \) when \( x = 4 \), find \( y \) when \( x = 5 \).

3. If \( x = 6 \) and \( y = kx + x \), then \( y = 18 \). What is the value of \( y \) when \( x = -5 \)?
Using the Rules of Exponents to Simplify

\( x^m \) is called an exponential expression. "\( x \)" is the base and "\( m \)" is the exponent. "\( m \)" tells how many times the base \( x \) is used as a factor \((x^3 = x \cdot x \cdot x)\)

### Rules

\[
x^m \cdot x^n = x^{m+n}
\]

You add exponents when you multiply powers of the same base.

\[
(x^m)^n = x^{mn}
\]

You multiply exponents when you raise a power to a power.

\[
\frac{x^m}{x^n} = x^{m-n}
\]

You subtract exponents when you divide powers.

\[
(xy)^m = x^m y^m
\]

Raise each factor inside the parentheses to the power outside.

\[
\left(\frac{x}{y}\right)^m = \frac{x^m}{y^m}
\]

Raise each factor in the quotient to the power outside.

\[
x^0 = 1
\]

Any number raised to the zero power equals 1.

\[
x^{-n} = \frac{1}{x^n}
\]

A negative exponent means take the reciprocal of the base to the positive power.

### Examples:

\[
x^3 \cdot x^2 = x^{3+2} = x^5
\]

\[
\left(\frac{x^4 y^5}{x^3 y^6}\right)^2 = (x^{4-3} y^{5-6})^2 = (xy^{-1})^2 = x^2 y^{-2} = \frac{x^2}{y^2}
\]

\[
(x^3)^2 = x^{3\cdot2} = x^6
\]

\[
\frac{x^5}{x^2} = x^{5-2} = x^3
\]

### Practice:

1. Simplify: \( \frac{(3y)^2}{9y} \)  
2. Simplify: \( -\frac{18x^4}{27x^3} \)

3. Simplify: \( (x^9)^2 \)  
4. Simplify: \( y^{-2} \cdot (y^4)^5 \)
5. Simplify: $2t^9 \cdot 8t^3$  
6. Simplify: $(x^2)^3 \cdot x^8$

7. Multiply: $(3a^2b^2)(-3a^2b^3)$  
8. Multiply: $(xy^2z)(x^2y^3z^2)$

9. Simplify: $(-4ab^3)(3a^{-2}b^4)$  
10. Simplify: $(2a^{-1}b^2)(4^{-1}ab^{-3})^2$
Using Scientific Notation

Scientific notation - the product of a number between 1 and 10 and a power of 10. 
\((a \times 10^n\), when \(1 \leq a < 10\))

Examples:

Write in scientific notation:

\[350 = 3.5 \times 100 = 3.5 \times 10^2\] Move the decimal 2 places to the right
\[.00275 = 2.75 \times 0.01 = 2.75 \times 10^{-3}\] Move the decimal 3 places to the left.

Write in decimal (standard) notation:

\[x \times 10^4 = 32000\] Move the decimal 4 places to the right (add zeros on right if needed)
\[6.5 \times 10^{-2} = .065\] Move the decimal 2 places to left (add zeros on left if needed)

Practice:

1. Write 0.0000356 in scientific notation: 2. Write 320,000 in scientific notation

3. Write 2.14 \times 10^{-3} in decimal notation. 4. Write 7.4 \times 10^5 in decimal notation.

5. Simplify: \(\frac{8 \times 10^{-2}}{2 \times 10^7}\) 6. Simplify: \(\frac{2.8 \times 10^5}{4 \times 10^{-3}}\)
Adding and Subtracting Polynomials

When you add or subtract polynomials, you add like terms, which are terms with the same variable and same exponent. $3x^2$ and $-5x^2$ are like terms, $3x$ and $2y$ are not like terms. Constant terms are like terms.

Change subtraction to addition of the opposite.

When you combine like terms, you add the coefficients.

Examples:

Add: $(3x^2 + 2x - 8) + (4x^2 - 3x + 7)$

$(3x^2 + 2x - 8) + (4x^2 - 3x + 7)$ Change subtraction to addition of opposites

$(3x^2 + 4x^2) + (2x - 3x) + (-8 + 7)$ Group like terms together

$7x^2 - 1x - 1$ Add like terms

$7x^2 - 1x - 1$ Rewrite using definition of subtraction

Subtract: $(2x^4 - 3x^2 + 7) - (5x^4 + 2x - 8)$

$(2x^4 + -3x^2 + 7) + - (5x^4 + 2x + -8)$ Change subtraction to addition of opposites

$(2x^4 + -3x^2 + 7) + (-5x^4 + -2x + 8)$ Change the signs of each term in 2nd parentheses

$(2x^4 + -5x^4) + -3x^2 + -2x + (7 + 8)$ Group like terms together.

$-3x^4 + -3x^2 + -2x + 15$ Add like terms

$-3x^4 + 3x^2 + 2x + 15$ Rewrite using the definition of subtraction

Practices:

1. Add: $(7x^2 - 5x + 6) + (-3x^2 + 4x - 10)$
2. Add: $(-3y^2 - 5y - 11) + (6y^2 - 3y)$

3. Subtract: $(x^2 - 2x + 7) - (3x^2 - 4x + 7)$
4. Subtract: $(4a^2 - 7a) - (-6a^2 + 5a - 7)$
**Multiplying Polynomials**

To multiply monomials, you multiply like factors. Remember to add the exponents of the like variables.

**Simplify:** \((3x^2y^4)(-4x^3y) = (3 \cdot -4)(x^2 \cdot x^3)(y^4 \cdot y) = -12x^5y^5\)

To multiply a monomial by a polynomial, you use the Distributive Property, \(a \cdot (b + c) = ab + bc\)

**Multiply:** \(3x(2x^3 - 4x + 5) = (3x \cdot 2x^3) - (3x \cdot 4x) + (3x \cdot 5) = 6x^4 - 12x^2 + 15x\)

To multiply two binomials, use FOIL. (\(F = \) first terms, \(O = \) outside terms, \(I = \) inside terms, and \(L = \) last terms.)

**Multiply:** \((x - 3)(2x + 5) = (x \cdot 2x) + (x \cdot 5) - (3 \cdot 2x) - (3 \cdot 5) = 2x^2 + 5x - 6x - 15 = 2x^2 - x - 15\)

In general, to multiply two polynomials, always multiply each term inside the first parentheses times each term inside the second parentheses.

**Practice:**

1. Simplify: \((-2a^2b^3)(-4ab^2)\)
2. Simplify: \((-a^2b)(6a^2b^3)\)
3. Multiply: \(-5y^2(3y - 4y^2)\)
4. Multiply: \(-3x^2(3x^2 - 2x - 6)\)
5. Multiply: \((2x + 5)(3x - 7)\)
6. Multiply: \((2x + 3)(4x - 9)\)
7. Multiply: \((x + 4)(x^2 - 4x + 16)\)
8. Multiply: \((3x + 5)(2x^2 - x - 1)\)
Factoring Polynomials: Greatest Common Factor

Look for the largest number that is a factor of the coefficients and constant term. Then, if the same variable is in every term, that variable to the lowest exponent is GCF for that variable. Write each term as the product of the GCF and the other factor. Rewrite in factored form using the Distributive Property: \( ab + ac = a(b + c) \)

Example:

Factor: \( 6a^2b^4 - 15a^4b \)  
Follow these steps:

The GCF for 6 and 15 is 3  
\( 6a^2b^4 - 15a^4b = \)

The GCF for \( a^2 \) and \( a^4 \) is \( a^2 \)  
\( 3a^2b \cdot 2b^3 - 3a^2b \cdot 5a^2 \)

The GCF for \( b^4 \) and \( b \) is \( b \)  
\( 3a^2b (2b^3 - 5a^2) \)

The GCF is \( 3a^2b \)

Practice:

1. Factor: \( 6a^2 - 2a \)  
2. Factor: \( 6x^4y^2 + 9x^3y \)

3. Factor: \( 8x^2 - 12x^3 + 16x^4 \)  
4. Factor: \( 8x^2y^2 - 12xy^2 + 20xy^3 \)
Factoring Polynomials: \( x^2 + bx + c \)

\( x^2 + bx + c \) is called a quadratic trinomial. If it can be factored, it can be factored as the product of two binomials.

Examples:
\[ x^2 + 5x + 6 = (x + 3)(x + 2) \quad \text{Factors of 6 are 2 and 3. Sum of 2 + 3 = 5 (coefficient of middle term)} \]
\[ x^2 - 4x + 3 = (x - 3)(x - 1) \quad \text{Factors of 3 whose sum is -4 are -3 and -1.} \]

When “c” is positive, then the signs in the binomials are the same as the sign of the middle term.

\[ x^2 - 2x - 3 = (x - 3)(x + 1) \quad \text{Factors of -3 whose sum is -2 are -3 and 1} \]
\[ x^2 + 2x - 8 = (x + 4)(x - 2) \quad \text{Factors of -8 whose sum is +2 are -4 and 2} \]

When “c” is negative, then the signs in the binomials are opposite of each other.

WATCH OUT: The sum of the factors must equal the coefficient of the middle term.

Use FOIL to check. Not all polynomials of the form \( x^2 + bx + c \) can be factored.

Practice:
1. Factor \( x^2 + 5x - 14 \)
2. Factor: \( m^2 - 4m - 21 \)
3. Factor: \( x^2 + 13x + 36 \)
4. Factor: \( y^2 - 21y + 80 \)
Factoring Polynomials: \( ax^2 + bx + c \)

The quadratic expression, \( ax^2 + bx + c \), can be factored as the product of two binomials as follows:

The factors of \( 3x^2 \) are
- Factors of 3x^2
- Factors of 2

\[ 3x^2 - 7x + 2 = (3x - 1)(x - 2) \]

The factors of 8x^2 are
- Factors of 8x^2
- Factors of -3

\[ 8x^2 + 2x - 3 = (4x + 3)(2x - 1) \]

The factors of 12x^2 are
- Factors of 12x^2
- Factors of 15

\[ 12x^2 + 28x + 15 = (6x + 5)(2x + 3) \]

Practice:

1. Factor: \( 3x^2 - 7x + 2 \)  
2. Factor: \( 6x^2 + 19x - 20 \)

3. Factor: \( 12a^2 + 19a + 4 \)  
4. Factor: \( 2a^2 - a - 15 \)
Factoring Polynomials: Difference Between Two Squares, Perfect Square Trinomials, of Sum and Difference Between Two Cubes

$a^2 - b^2$ is called the difference between two squares and can be factored as the product of the sum and difference of two binomials as follows:

$$a^2 - b^2 = (a + b)(a - b)$$

Examples: $x^2 - 25 = (x + 5)(x - 5)$  $9x^2 - 4 = (3x + 2)(3x - 2)$

$a^2 + 2ab + b^2$ and $a^2 - 2ab + b^2$ are called perfect square trinomials and can be factored as the square of a binomial as follows:

$$a^2 + 2ab + b^2 = (a + b)^2$$
$$a^2 - 2ab + b^2 = (a - b)^2$$

Examples: $4x^2 - 12x + 9 = (2x - 3)^2$  $16x^2 + 8x + 1 = (4x + 1)^2$

$a^3 + b^3$ and $a^3 - b^3$ are called the sum and difference between two cubes and can be factored as follows:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Examples: $x^3 + 8 = (x + 2)(x^2 - 2x + 4)$  $27x^3 - 64 = (3x - 4)(9x^2 + 12x + 16)$

Practice:

1. Factor: $x^2 - 9$  
2. Factor: $25y^2 - 81$
3. Factor: $9a^2 - 24a + 16$  
4. Factor: $25x^2 + 30x + 9$
5. Factor: $x^3 - 125$  
6. Factor: $27y^3 + 64$
7. Factor $x^2 + 25$  
8. Factor: $a^2 + 9a + 9$
**Solving Quadratic Equations Using Factoring**

Steps to follow are:
1. Simplify and set equal to 0. Combine like terms.
2. Arrange in the following order: \( ax^2 = bx + c = 0 \)
3. Factor. Look for GCF first. Then factor as the product of binomials.
4. Set each factor equal to 0 and solve these equations.

Examples:

\[
x^2 - 5x - 6 = 0 \quad \quad \quad \quad 3x^2 - 6x = 0
\]
\[
(x - 6)(x + 1) = 0 \quad \quad \quad \quad 3x(x - 2) = 0
\]
\[
x - 6 = 0 \text{ or } x + 1 = 0 \quad \quad \quad 3x = 0 \text{ or } x - 2 = 0
\]
\[
x = 6 \text{ or } x = -1 \quad \quad \quad x = 0 \text{ or } x = 2
\]
\[
2(x^2 + 1) = 3x + 1 \quad \quad \quad x(x + 3) = 28
\]
\[
2x^2 + 2 = 3x + 1 \quad \quad \quad x^2 + 3x = 28
\]
\[
2x^2 - 3x + 1 - 0 \quad \quad \quad x^2 + 3x - 28 = 0
\]
\[
(2x - 1)(x - 1) = 0 \quad \quad \quad (x + 7)(x - 4) = 0
\]
\[
2x - 1 = 0 \text{ or } x - 1 = 0 \quad \quad \quad x + 7 = 0 \text{ or } x - 4 = 0
\]
\[
x = \frac{1}{2} \text{ or } x = 1 \quad \quad \quad x = -7 \text{ or } x = 4
\]

Practice:

1. Solve: \( x^2 - 3x = 0 \)
2. Solve: \( y^2 + 5y = 0 \)
3. Solve: \( t^2 - 2t = 15 \)
4. Solve: \( 2x^2 - 5x = 3 \)
5. Solve: \( x(x + 8) = x + 18 \)
6. Solve: \( x(x - 3) = x + 12 \)
Applying Quadratic Equations

Examples:

The sum of the square of a number and four times the number is thirty-two. Find the number.

Let \( x \) = the number

Equation: \( x^2 + 4x = 32 \)

\( x^2 = \text{square of } x \)

\( 4x = \text{4 times } x \)

\( (x - 4)(x + 8) = 0 \)

\( x - 4 = 0 \) or \( x + 8 = 0 \)

\( x = 4 \) or \( x = -8 \)

The base of a triangle is 8cm more than its height. If the area (\( A = \frac{1}{2} bh \)) of the triangle is 24cm\(^2\), find the base and height.

\( h = \text{height} \)

Formula: \( A = \frac{1}{2} bh \)

\( h + 8 = \text{base} \)

\( 24 = \frac{1}{2} (h + 8) h \)

\( 2(24) = 2(1/2) (h^2 + 8h) \)

\( 48 = h^2 + 8h \)

\( 0 = h^2 + 8h - 48 \)

\( 0 = (h + 12)(h - 4) \)

\( h + 12 = 0 \) or \( h - 4 = 0 \)

\( h = -12 \) or \( 4 \)

REMINDER: Read the problem carefully.
Define the variables.
Write an equation that relates the variables.
Solve and check. Make sure you answer the question.
Since \( h \) represents the height, then \( h = 4 \). Measurement is always positive. The height is 4 cm and the base is 12 cm.

Practice:

1. The sum of a number and its square is 30. Find the number.

2. The difference between the square of a number and 3 times the number is 40. Find the number.

3. The length of a rectangle is 2 cm more than twice the width. The area of the rectangle is 60 cm\(^2\). Find the dimensions of the rectangle.

4. The length of a rectangle is 9 in. and width is 6 in. If both the length and the width are increased by equal amounts, the area of the rectangle is doubled. Find the dimensions of the larger rectangle.
Simplifying Rational Expressions

You reduce rational expressions by dividing out like factors in the numerator and denominator.

Examples:

Simplify: \( \frac{8x^3y^5}{2x^2y} = \frac{2x^2y \cdot 4xy^4}{2x^2y \cdot 1} = 4xy^4 \)

Reduce: \( \frac{20x^2y^4}{15x^4y^2} = \frac{5x^2y^2 \cdot 4y^2}{5x^2y^2 \cdot 3x^2} = \frac{4y^2}{3x^2} \)

Simplify: \( \frac{x^2 - 3x}{x^2 - 2x - 3} = \frac{x(x - 3)}{(x + 1)(x - 3)} = \frac{x}{x + 1} \)

Practice:

1. Simplify: \( \frac{50x^3y^6}{45xy^9} \)
2. Simplify: \( \frac{32a^4b^{12}}{24a^8b^7} \)
3. Simplify: \( \frac{a^2 + 6a}{a^2 + 7a + 6} \)
4. Simplify: \( \frac{x^2 - 3x - 10}{x^2 - 6x + 5} \)
Multiplying and Dividing Rational Expressions

A rational expression is an algebraic expression that is a fraction. Use the rules for multiplying and dividing fractions to do these problems.

When you multiply monomial rational expressions, multiply the numerators and the denominators. Then reduce to lowest terms.

\[
\frac{3xy}{2a^2} \cdot \frac{8y^2}{6ax^2} = \frac{24xy^3}{12a^3x^2} = \frac{2y^3}{a^3x}
\]

When you multiply polynomial rational expressions, first factor each numerator and denominator completely. Divide out like factors in the numerators and denominators. Then write the product in factored form.

\[
\frac{x^2 - 2x - 3}{x^2 + 4x} \cdot \frac{x^2 - 16}{x + 1} = \frac{(x - 3)(x + 1)(x + 4)(x - 4)}{x(x + 4)(x + 1)} = \frac{(x - 3)(x - 4)}{x}
\]

When you divide, you multiply by the reciprocal of the divisor (expression that follows the division sign.) Then follow the rules for multiplication.

\[
\frac{x^2 - 2x}{x - 3} \div \frac{x}{x^2 - 9} = \frac{x^2 - 2x}{x - 3} \cdot \frac{x}{x - 3} = \frac{x(x - 2)}{x - 3} \cdot \frac{x + 3(x - 3)}{x} = (x - 2)(x + 3)
\]

Practice:

1. Multiply: \( \frac{x^2 - x - 20}{x^2y^3} \cdot \frac{x^3y^2}{x^2 - 10x + 25} \)

2. Multiply: \( \frac{x^2 - 3x - 10}{x^2 - 5x} \cdot \frac{x + 2}{x^2 - 4} \)

3. Divide: \( \frac{x^2 + 2x}{x^2 - 4} \div \frac{x + 2}{x - 2} \)

4. Divide: \( \frac{x^2 + 3x + 2}{xy^2} \div \frac{x^2 + 4x + 4}{x^2y^2} \)
Simplifying Radical Expressions

The symbol $\sqrt{\cdot}$ is called the radical sign and is used to denote the square root of a number. This is the opposite operation of squaring a number.

$\sqrt{16} = 4$ because $4^2 = 16$

The symbol, $\sqrt[3]{\cdot}$, denotes the cube root of a number. $\sqrt[3]{8} = 2$ since $2^3 = 8$.

Likewise, $\sqrt[3]{a^3} = a$ and $\sqrt[3]{a^3} = a$.

When simplifying a radical expression (square root), we look for perfect square factors. The integers between 0 and 200 that are perfect squares are:

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, and 196.

Examples:

\[
\begin{align*}
\sqrt{49} &= 7 & \sqrt{50} &= \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5\sqrt{2} \\
(49 \text{ is a perfect square}) & & (25 \text{ is the largest perfect square factor of } 50)
\end{align*}
\]

\[
\begin{align*}
\sqrt{25a^2b^3} &= 5ab^2 & \text{(Each of the factors are perfect squares. } b^4 = (b^2)^2)\\
\sqrt{20a^2b^3} &= \sqrt{4 \cdot 5 \cdot a^2 \cdot b^2 \cdot b} = \sqrt{4a^2b^2} \cdot \sqrt{5b} = 2ab\sqrt{5b}
\end{align*}
\]

When simplifying a cube root, $\sqrt[3]{\cdot}$, we look for perfect cube factors. The first eight perfect cubes are:

1, 8, 27, 63, 125, 343, and 512.

Example:

\[
\begin{align*}
\sqrt[3]{12} &= 5 & \sqrt[3]{24a^2b^7} &= \sqrt[3]{8a^2b^6 \cdot 3a^2b} = \sqrt[3]{8a^2b^6} \cdot \sqrt[3]{3a^2b} = 2ab^2\sqrt[3]{3a^2b}
\end{align*}
\]

REMEMER: $a^n$ is a perfect square if the exponent, $n$, is an even number and $a^n$ is a perfect cube if the exponent, $n$, is divisible by 3.

Practice:

1. Simplify: $\sqrt{169}$
2. Simplify: $\sqrt[3]{9a^4b^{10}}$
3. Simplify: $\sqrt{180}$

4. Simplify: $\sqrt{250}$

5. $\sqrt{95}$ is between what two integers?
Adding, Subtracting, Multiplying and Dividing Radical Expressions

When you add or subtract two radical expressions, you must have the same value or expression under the radical and both must have the same root (index). Then you add or subtract the values in front of the radicals (coefficients).

Examples:

\[3\sqrt{5} + 8\sqrt{5} - 6\sqrt{5} = 5\sqrt{5}\]
\[2\sqrt{3a} - 6\sqrt{3a} + 4\sqrt{3a} = 6\sqrt{3a} - 6\sqrt{3a}\]

When you multiply or divide radical expressions, multiply or divide the expressions under the radicals and the expressions in front of the radical separately. Remember, both must have a common root, but do not have to have the same expression under the radical

Examples:

\[\sqrt{18} \cdot \sqrt{2} = \sqrt{18 \cdot 2} = \sqrt{36} = 6\]
\[3\sqrt{5} \cdot 4\sqrt{2} = 3 \cdot 4\sqrt{5 \cdot 2} = 12\sqrt{10}\]

Practice:

1. Simplify: \(3\sqrt{2} - 7\sqrt{2}\)  
2. Simplify: \(\sqrt{32} + \sqrt{50}\)

3. Multiply: \(\sqrt{8} \cdot \sqrt{2}\)  
4. Multiply: \(\sqrt{6} \cdot \sqrt{15}\)
### Answer Key

**Using Order of Operations**

1. -2  
2. -7  
3. 8  
4. -6

**Simplifying Complex Fraction**

1. -96/17  
2. 1 5/11  
3. 7/12  
4. -1/3

**Evaluating Algebraic Expressions**

1. 7  
2. 7  
3. -16  
4. -13  
5. 7  
6. 9

**Using Formulas**

1. 20 ft.  
2. 62.8 in.  
3. 18.3 yds²  
4. 226.08 ft²

**Solving 1-Variable Equations**

1. 6  
2. 4  
3. 5
4. \( x > 4 \)
5. \( x > 5 \)
6. \( x \leq -8/5 \)
7. \( x \geq -2 \)

**Rewriting Verbal Sentences**
1. \(-9n -6\)
2. \(\frac{10}{n + 7}\)
3. \(30 - g\)
4. \(4w + 25\)
5. \(4n - 5 = 3\)
6. \(4\)
7. \(5, 7, 9, 11 \text{ or } 7, 9, 11, 13, \text{ or } 9, 11, 13, 15\)
8. \(10n + 1200 \leq 2350\)

**Solving for Particular Variable**
1. \(a = \frac{R - C}{P}\)
2. \(t = \frac{k}{r - v}\)
3. \(a = \frac{cR + T}{B}\)
4. \( R = 2S - T \)
5. \( c - ax \)

Solving Problems Involving "k"

1. 10
2. 3
3. -15

Using the Rules for Exponents

1. \( y \)
2. \( \frac{-2x}{3} \)
3. \( x^{18} \)
4. \( y^{18} \)
5. \( 16t^{12} \)
6. \( x^{14} \)
7. \( -9a^4b^5 \)
8. \( x^3y^5z^3 \)
9. \( \frac{-12b}{a} \)
10. \( \frac{a}{8b^4} \)

Using Scientific Notation

1. \( 3.56 \times 10^{-5} \)
2. \( 3.2 \times 10^5 \)
3. \( 0.00214 \)
4. \( 740,000 \)
5. \( 4 \times 10^{-9} \)
6. \( 7 \times 10^7 \)
<table>
<thead>
<tr>
<th>Adding/Subtracting Polynomials</th>
<th>1. $4x^2 - x - 4$</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>2. $3y^2 - 8y - 11$</td>
</tr>
<tr>
<td></td>
<td>3. $-2x^2 + 2x$</td>
</tr>
<tr>
<td></td>
<td>4. $10a^2 - 12a + 7$</td>
</tr>
<tr>
<td>Multiplying Polynomials</td>
<td>1. $8a^3b^5$</td>
</tr>
<tr>
<td></td>
<td>2. $-6a^4b^4$</td>
</tr>
<tr>
<td></td>
<td>3. $-15y^3 + 20y^4$</td>
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<td></td>
<td>4. $-9x^4 + 6x^3 + 18x^2$</td>
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<td>5. $6x^2 + x - 35$</td>
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<td>6. $8x^2 - 6x - 27$</td>
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<td></td>
<td>7. $x^3 + 64$</td>
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<td></td>
<td>8. $6x^3 + 7x^2 - 8x - 5$</td>
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<tr>
<td>Factoring GCF</td>
<td>1. $2a(3a - 1)$</td>
</tr>
<tr>
<td></td>
<td>2. $3x^3y (2xy + 3)$</td>
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<tr>
<td></td>
<td>3. $4x^2(2 - 3x + 4x^2)$</td>
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<td></td>
<td>4. $4xy^2 (2x - 3 + 5y)$</td>
</tr>
<tr>
<td>Factoring $x^2 + bx + c$</td>
<td>1. $(x + 7)(x - 2)$</td>
</tr>
<tr>
<td></td>
<td>2. $(m - 7)(m + 3)$</td>
</tr>
<tr>
<td></td>
<td>3. $(x + 9)(x + 4)$</td>
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<tr>
<td></td>
<td>4. $(y - 16)(y - 5)$</td>
</tr>
<tr>
<td>Factoring $ax^2 + bx + c$</td>
<td>1. $(3x - 1)(x - 2)$</td>
</tr>
<tr>
<td></td>
<td>2. $(6x - 5)(x + 4)$</td>
</tr>
</tbody>
</table>
3. \((4a + 1)(3a + 4)\)

4. \((a - 3)(2a + 5)\)

Special Factoring

1. \((x + 3)(x - 3)\)

2. \((5y + 9)(5y - 9)\)

3. \((3a - 4)^2\)

4. \((5x + 3)^2\)

5. \((x - 5)(x^2 + 5x + 25)\)

6. \((3y + 4)(9y^2 - 12y + 16)\)

7. non-factorable

8. non-factorable

Solving Quadratic Equations

1. 0 or 3

2. 0 or -5

3. 5, -3

4. 3, -1/2

5. -9, 2

6. -2, 6

Applying Quadratic Equations

1. -6 or 5

2. 8 or -5

3. 5 cm x 12 cm

4. 12 in x 9 in

Simplifying Rational Expressions

1. \(\frac{10x^2}{9y^3}\)
2. $\frac{4b^5}{3a^4}$

3. $\frac{a}{a + 1}$

4. $\frac{x + 2}{x - 1}$

**Mult/Div Rational Expressions**

1. $\frac{x(x + 4)}{y(x - 5)}$

2. $\frac{x + 2}{x(x - 2)}$

3. $\frac{x}{x + 2}$

4. $\frac{x(x + 1)}{x + 2}$

**Simplifying Radical Expressions**

1. 13

2. $3a^2 b^5$

3. $6\sqrt{5}$

4. $5\sqrt{10}$

5. 9 and 10

**Adding, Subt, Mult, Div., Radicals**

1. $-4\sqrt{2}$

2. $9\sqrt{2}$

3. 4

4. $3\sqrt{10}$